

Frame Dragging

Albert Einstein's theory of general relativity predicts that rotating bodies drag spacetime around themselves in a phenomenon referred to as frame-dragging. The rotational frame-dragging effect was first derived from the theory of general relativity in 1918 by the Austrian physicists Josef Lense and Hans Thirring, and is also known as the Lense–Thirring effect. Lense and Thirring predicted that the rotation of an object would alter space and time, dragging a nearby object out of position compared with the predictions of Newtonian physics. The predicted effect is small, only about one part in a few trillion. To detect it, it is necessary to examine a very massive object, or build an instrument that is very sensitive. The field of study specific to the subject of field effects caused by moving matter is known as gravitomagnetism.

Rotational frame-dragging appears in the general principle of relativity and similar theories in the vicinity of rotating massive objects. Under the Lense–Thirring effect, the frame of reference in which a clock ticks the fastest is one which is rotating around the object as viewed by a distant observer. This also means that light traveling in the direction of rotation of the object will move around the object faster than light moving against the rotation as seen by a distant observer. It is now the best-known effect, partly thanks to the Gravity Probe B experiment. This article should be digested as a general reference to the Time article on the Quantum Physics page of this site.

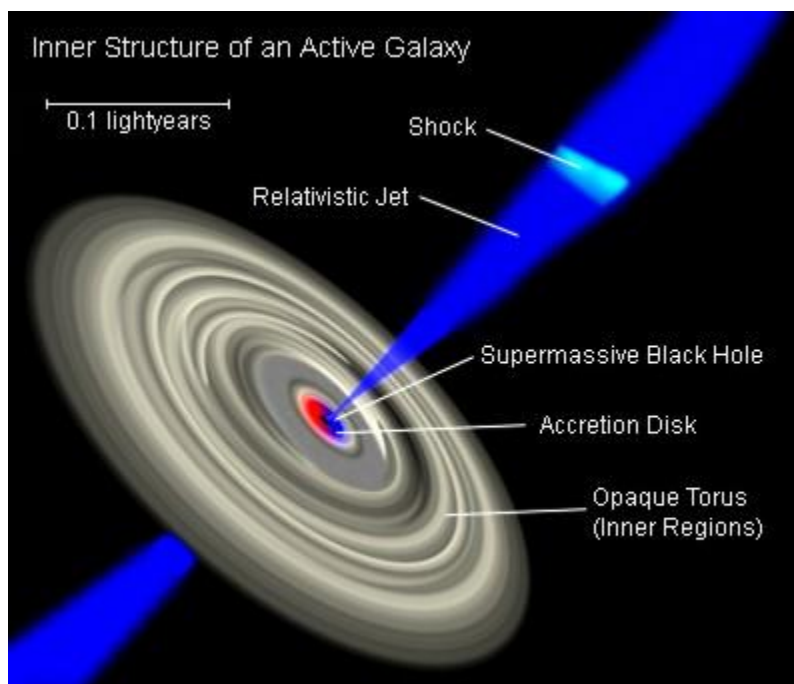
Linear frame dragging is the similarly inevitable result of the general principle of relativity, applied to linear momentum. Although it arguably has equal theoretical legitimacy to the "rotational" effect, the difficulty of obtaining an experimental verification of the effect means that it receives much less discussion and is often omitted from articles on frame-dragging. Static mass increase is a third effect noted by Einstein. The effect is an increase in inertia of a body when other masses are placed nearby. While not strictly a frame dragging effect (the term frame dragging is not used by Einstein), it is demonstrated by Einstein to derive from the same equation of general relativity. It is also a tiny effect that is difficult to confirm experimentally.

Relativistic jets provide evidence for the reality of frame-dragging. According to Wei Cui, S. N. Zhang, and Wan Chen, in the *The Astrophysical Journal*, 492:L53–L57, 1998 January 1, in their paper, "EVIDENCE FOR FRAME-DRAGGING AROUND SPINNING BLACK HOLES IN X-RAY BINARIES" they write:

"In the context of black hole spin in X-ray binaries, we propose that certain types of quasi-periodic oscillations (QPOs) observed in the light curves of black hole binaries (BHBs) are produced by X-ray modulation at the precession frequency of accretion disks, because of relativistic dragging of inertial frames around spinning black holes. These QPOs tend to be relatively stable in their centroid frequencies. They have been observed in the frequency range of a few hertz to a few hundred hertz for several black holes with dynamically determined masses. By comparing the computed disk precession frequency with that of the observed QPO, we can derive the black hole spin, given its mass. When applying this model to GRO J1655240, GRS 19151105, Cyg X-1,

and GS 1124268, we found that the black holes in GRO J1655240 and GRS 19151105, the only known BHBs that occasionally produce superluminal radio jets, spin at a rate close to the maximum limit, while Cyg X-1 and GS 1124268, typical (persistent and transient) BHBs, contain only moderately rotating ones. Extending the model to the general population of black hole candidates, the fact that only low-frequency QPOs have been detected is consistent with the presence of only slowly spinning black holes in these systems. Our results are in good agreement with those derived from spectral data, thus strongly supporting the classification scheme that we proposed previously for BHBs."

<http://www.physics.purdue.edu/astro/CuiPapers/Evidence%20for%20Frame%20Dragging%20around%20Spinning%20Black%20Holes%20in%20X-Ray%20Binaries.pdf>



Relativistic Jet.

The environment around the AGN where the relativistic plasma is collimated into jets which escape along the pole of the supermassive black hole

Frame-dragging may be illustrated most readily using the Kerr metric, which describes the geometry of spacetime in the vicinity of a mass M rotating with angular momentum J

$$c^2 d\tau^2 = \left(1 - \frac{r_s r}{\rho^2}\right) c^2 dt^2 - \frac{\rho^2}{\Lambda^2} dr^2 - \rho^2 d\theta^2 - \left(r^2 + \alpha^2 + \frac{r_s r \alpha^2}{\rho^2} \sin^2 \theta\right) \sin^2 \theta d\phi^2 + \frac{2r_s r \alpha c \sin^2 \theta}{\rho^2} d\phi dt$$

where r_s is the Schwarzschild radius

$$r_s = \frac{2GM}{c^2}$$

and where the following shorthand variables have been introduced for brevity

$$\alpha = \frac{J}{Mc}$$

$$\rho^2 = r^2 + \alpha^2 \cos^2 \theta$$

$$\Lambda^2 = r^2 - r_s r + \alpha^2$$

In the non-relativistic limit where M (or, equivalently, r_s) goes to zero, the Kerr metric becomes the orthogonal metric for the oblate spheroidal coordinates

$$c^2 d\tau^2 = c^2 dt^2 - \frac{\rho^2}{r^2 + \alpha^2} dr^2 - \rho^2 d\theta^2 - (r^2 + \alpha^2) \sin^2 \theta d\phi^2$$

We may re-write the Kerr metric in the following form

$$c^2 d\tau^2 = \left(g_{tt} - \frac{g_{t\phi}^2}{g_{\phi\phi}} \right) dt^2 + g_{rr} dr^2 + g_{\theta\theta} d\theta^2 + g_{\phi\phi} \left(d\phi + \frac{g_{t\phi}}{g_{\phi\phi}} dt \right)^2$$

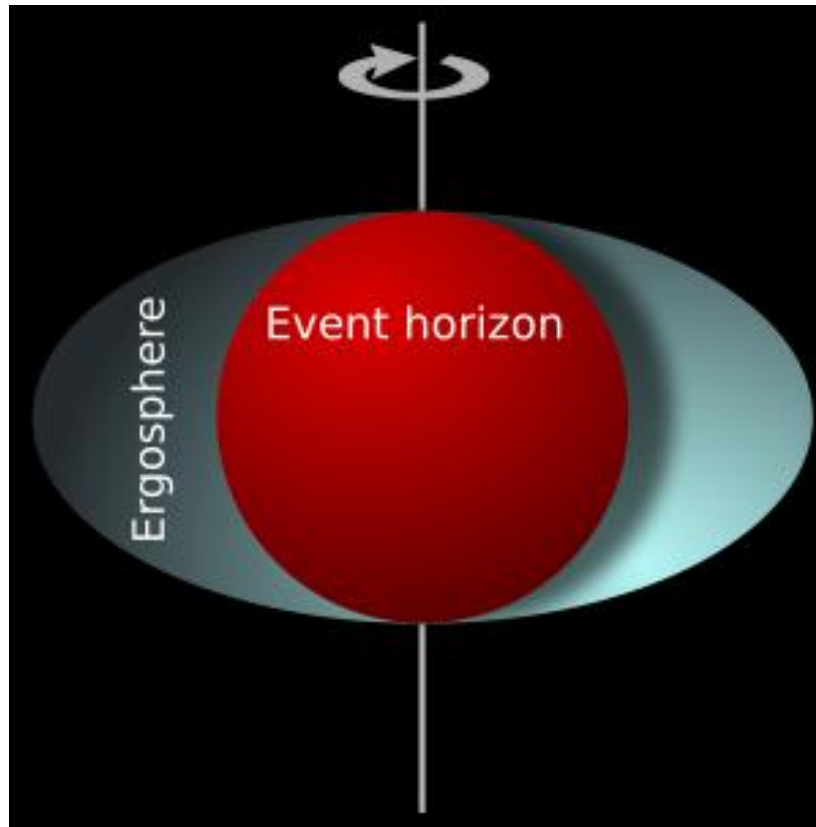
This metric is equivalent to a co-rotating reference frame that is rotating with angular speed Ω that depends on both the radius r and the colatitude θ

$$\Omega = -\frac{g_{t\phi}}{g_{\phi\phi}} = \frac{r_s \alpha r c}{\rho^2 (r^2 + \alpha^2) + r_s \alpha^2 r \sin^2 \theta}$$

In the plane of the equator this simplifies to:

$$\Omega = \frac{r_s \alpha c}{r^3 + \alpha^2 r + r_s \alpha^2}$$

Thus, an inertial reference frame is entrained by the rotating central mass to participate in the latter's rotation; this is frame-dragging.



The two surfaces on which the Kerr metric appears to have singularities; the inner surface is the spherical event horizon, whereas the outer surface is an oblate spheroid. The ergosphere lies between these two surfaces; within this volume, the purely temporal component g_{tt} is negative, i.e., acts like a purely spatial metric component. Consequently, particles within this ergosphere must co-rotate with the inner mass, if they are to retain their time-like character.

An extreme version of frame dragging occurs within the ergosphere of a rotating black hole. The Kerr metric has two surfaces on which it appears to be singular. The inner surface corresponds to a spherical event horizon similar to that observed in the Schwarzschild metric; this occurs at

$$r_{inner} = \frac{r_s + \sqrt{r_s^2 - 4\alpha^2}}{2}$$

where the purely radial component g_{rr} of the metric goes to infinity. The outer surface is not a sphere, but an oblate spheroid that touches the inner surface at the poles of the rotation axis, where the colatitude θ equals 0 or π ; its radius is defined by the formula

$$r_{outer} = \frac{r_s + \sqrt{r_s^2 - 4\alpha^2 \cos^2 \theta}}{2}$$

where the purely temporal component g_{tt} of the metric changes sign from positive to negative. The space between these two surfaces is called the ergosphere. A moving particle experiences a positive proper time along its worldline, its path through spacetime. However, this is impossible within the ergosphere, where g_{tt} is negative, unless the particle is co-rotating with the interior mass M with an angular speed at least of Ω . However, as seen above, frame-dragging occurs about every rotating mass and at every radius r and colatitude θ , not only within the ergosphere.

Lense–Thirring effect inside a rotating shell

Inside a rotating spherical shell the acceleration due to the Lense–Thirring effect would be

$$\bar{a} = -2d_1 (\bar{\omega} \times \bar{v}) - d_2 [\bar{\omega} \times (\bar{\omega} \times \bar{r}) + 2 (\bar{\omega} \bar{r}) \bar{\omega}]$$

where the coefficients are

$$d_1 = \frac{4MG}{3Rc^2}$$

$$d_2 = \frac{4MG}{15Rc^2}$$

for $MG \ll Rc^2$ or more precisely,

$$d_1 = \frac{4\alpha(2 - \alpha)}{(1 + \alpha)(3 - \alpha)}, \quad \alpha = \frac{MG}{2Rc^2}$$

The space-time inside the rotating spherical shell will not be flat. A flat space-time inside a rotating mass shell is possible if the shell is allowed to deviate from a precisely spherical shape and the mass density inside the shell is allowed to vary.

We don't encounter more frame dragging because what we perceive of as "space" isn't all there is, the particles are oscillating between different energy levels as they move between adjacent universes/subuniverses. In the end, these might provide the mechanism for why the laws of physics and constants are what they are in any particular universe/subuniverse (this includes ZPE density.)