

## The Lunar Equation

### Julian Day Numbers

The Julian Day Count is a uniform count of days from a remote epoch in the past, specifically -4712 January 1, 12 hours Greenwich Mean Time (Julian proleptic Calendar). At this instant, the Julian Day Number is 0. It is convenient for astronomers to use since it is not necessary to worry about odd numbers of days in a month, leap years, etc. Once you have the Julian Day Number of a particular date in history, it is easy to calculate time elapsed between it and any other Julian Day Number.

The Julian Day Count has nothing to do with the Julian Calendar introduced by Julius Caesar. It is named for Julius Scaliger, the father of Josephus Justus Scaliger, who invented the concept. It can also be thought of as a logical follow-on to the old Egyptian civil calendar, which also used years of constant lengths.

Scaliger chose the particular date in the remote past because it was before recorded history and because in that year, three important cycles coincided with their first year of the cycle: The 19-year Metonic Cycle, the 15-year Indiction Cycle (a Roman Taxation Cycle) and the 28-year Solar Cycle (the length of time for the old Julian Calendar to repeat exactly).

It is easy (with your calculator) to calculate the Julian Day Number of any date given on the Gregorian Calendar. The Julian Day Number so calculated will be for 0 hours, GMT, on that date. Here's how to do it:

- 1) Express the date as Y M D, where Y is the year, M is the month number (Jan = 1, Feb = 2, etc.), and D is the day in the month.
- 2) If the month is January or February, subtract 1 from the year to get a new Y, and add 12 to the month to get a new M. (Thus, we are thinking of January and February as being the 13th and 14th month of the previous year).
- 3) Dropping the fractional part of all results of *all multiplications and divisions*, let
$$A = Y/100$$
$$B = A/4$$
$$C = 2-A+B$$
$$E = 365.25x(Y+4716)$$
$$F = 30.6001x(M+1)$$
$$JD= C+D+E+F-1524.5$$

This is the Julian Day Number for the beginning of the date in question at 0 hours, Greenwich Time. Note that this always gives you a half day extra. That is because the Julian Day begins at *noon*, Greenwich Time. While this is very convenient for astronomers (who until recently only observed at night), it is

confusing.

Example: If the date is 1582 October 15,

Y = 1582  
M = 10  
D = 15  
A = 15  
B = 3  
C = -10  
E = 2300344  
F = 336  
JD = 2299160.5

To convert a Julian Day Number to a Gregorian date, assume that it is for 0 hours, Greenwich Time (so that it ends in 0.5). Do the following calculations, again dropping the fractional part of all multiplications and divisions. *Note: This method will not give dates accurately on the Gregorian Proleptic Calendar, i.e., the calendar you get by extending the Gregorian calendar backwards to years earlier than 1582, using the Gregorian leap year rules. In particular, the method fails if  $Y < 400$ .*

Z = JD+0.5  
W = (Z - 1867216.25)/36524.25  
X = W/4  
A = Z+1+W-X  
B = A+1524  
C = (B-122.1)/365.25  
D = 365.25xC  
E = (B-D)/30.6001  
F = 30.6001xE  
Day of month = B-D-F  
Month = E-1 or E-13 (must get number less than or equal to 12)  
Year = C-4715 (if Month is January or February) or C-4716 (otherwise)

Example: Check the first calculation by starting with JD = 2299160.5

Z = 2299161  
W = 11  
X = 2  
A = 2299171  
B = 2300695  
C = 6298  
D = 2300344  
E = 11  
F = 336

Day of Month = 15  
Month = 10  
Year = 1582

*Practice:* Calculate the Julian Day Number for 0 hours GMT on the date of your birthday. Check your result by calculating the date of your birthday from the Julian Day Number.

*Remark:* You can do the calculation of Julian Calendar Date-->Julian Day Number by ignoring the calculation of A and B, and setting C=0; to go from Julian Day Number to Julian Calendar Date, bypass the calculation of W and X and simply set A=Z. These calculations are useful also when converting between the Gregorian and Julian calendars (e.g., to correlate dates on the Julian Calendar in England prior to 1752 with dates on the Gregorian calendar). For example, to go from Gregorian to Julian calendar date, convert the Gregorian date to Julian Day Number, then convert the Julian Day Number to Julian calendar date. This method even works for dates prior to 1582 and correctly gives years prior to the Common Era as negative years (with year 0 corresponding to 1 BCE, year -1 corresponding to 2 BCE, etc.) However, it does *not* work with negative Julian Day Numbers and does *not* work when going to the Gregorian calendar for years before 400 CE.

### Lunation

The mean time for one lunar phase cycle (i.e., the synodic period of the Moon) is 29.530589 days, or 29 days, 12 hours, 44 minutes, 3 seconds. More specifically, a lunation is also commonly defined as the mean time between successive new moons. However, as a result of torques from the Sun, the actual time interval between consecutive new moons varies greatly. Meeus (1988) gives a table of the shortest (29 days 06 hours 35 minutes) and longest (29 days 19 hours 55 minutes) lunations from 1900 to 2100.

New moons occur on Julian days

$$2449128.59 + 29.53058867n \pm 0.25,$$

where  $n$  is an integer. The lunation number is defined such that lunation  $n = 1$  corresponds to the first new moon that occurred in 1923.